## Quiz 8-6 November 2019

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).
Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

| Problem | Weight | Score |
| :---: | :---: | :---: |
| 1 | 1 |  |
| 2 | 1 |  |
| 3 | 1 |  |
| 4 | 1 |  |
| 5 | 1 |  |
| Total |  | $/ 50$ |

For Problems 1-4, let

$$
f(x, y, z)=x y+x z+2 y z+\frac{1}{x}
$$

Problem 1. Find the gradient of $f$.

Problem 2. Find the Hessian of $f$.

Problem 3. $\left(-1, \frac{1}{2}, \frac{1}{2}\right)$ is a critical point of $f$. Find the principal minors of $f$ at $\left(-1, \frac{1}{2}, \frac{1}{2}\right)$.

Problem 4. Classify the critical point $\left(-1, \frac{1}{2}, \frac{1}{2}\right)$ as a local minimum, local maximum, or saddle point. Briefly explain why.

Problem 5. Suppose a company sells one product in two markets. Let

$$
\begin{array}{lll}
Q_{1} & =\text { number of units produced for market } 1 & \\
P_{1} & =\text { unit price in market } 1 & \\
R_{1} & =P_{1} Q_{1}=\text { number of units produced for } \mathrm{n} \\
C & =\text { cost of production market } 1 &
\end{array}
$$

Assume $Q_{1}=-2 P_{1}+40, Q_{2}=-3 P_{2}+48, C=10\left(Q_{1}+Q_{2}\right)$.
Suppose the company decides to sell in both markets at the same time. Write the profit as a function of $Q_{1}$ and $Q_{2}$, the number of units produced in markets 1 and 2. Do not find the maximum profit.

